THE ENERGY CALIBRATION AND RESOLUTION OF THE ENERGY SPECTROMETER LU-224

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1 The Energy Calibration of the Energy Spectrometer

The energy spectrometer line is shown in Fig.1 [1]. The magnetic field based on some experimental points is shown in Fig.2. For the equivalent deflection an effective field edge can be assumed at

$$z^* = \int_0^\infty \frac{B(z)\,dz}{B_o} \tag{1}$$

supposing $z = 0$ is in the uniform field region. According to the calculated data, Δz =2.85 cm.

Since the transverse dimension of the uniform region (~7 cm) is much larger than the deviation distance of the actual trajectory from the main axis, Δx_{max}, as shown in Fig.3,

$$\Delta x_{max} = R_o - \left(R_o - \frac{\Delta z}{\sin\frac{\phi}{2}}\right) = \Delta z \tan\frac{\phi}{4} \sim 0.5\,cm \tag{2}$$

the beam deflection is the same as that in the ideal uniform field with the hard effective edge. If both side of magnet is symmetry, it can be deduced from ΔABO' ~ ΔACO that the actual curved radius is determined by the following equation:

$$R = R_o + \Delta z \cdot \cot\frac{\phi}{2} \tag{3}$$

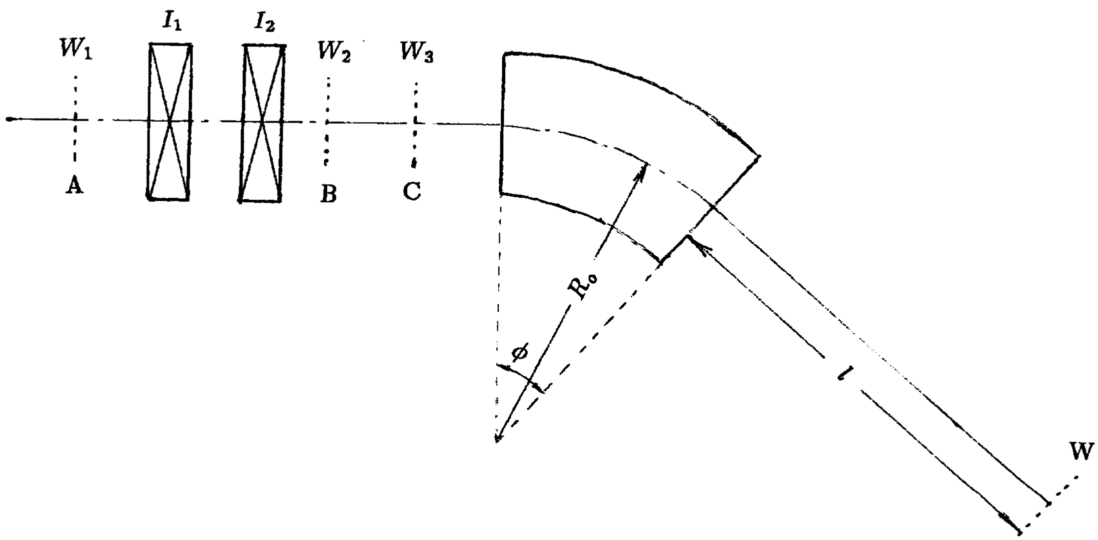

Figure 1: The energy measurement line

In our case, R_o = 4.25 m, ϕ = 40°, so R =4.328 m. It is said that the beam energy should be calculated by:

$$W = \sqrt{W_o^2 + 9 \times 10^4 R^2 B^2} - W_o \tag{4}$$

instead of

$$W = \sqrt{W_o^2 + 9 \times 10^4 R_o^2 B^2} - W_o \tag{5}$$

As an estimation, if the beam energy is 401.5 MeV, according to Eq.(5) to calculate, the beam energy would be : W =389.3 MeV. That means the error is $\sim$ 3.04%. Thus the correction of the spraying field is not negligible. The field given in Fig.2 is based on the theoretical calculation fitted to some experimental values. If more accurate result is required, the further experimental measurement may be required.

The other important factor effecting on the energy calibration is the beam centering with the axis. The effect of this deviation can be estimated by the following equation:

$$\frac{\Delta p}{p} \doteq \frac{\Delta x_t}{M_{13}} = \frac{1}{M_{13}} \left[\Delta x_o (\cos\phi - \frac{L}{R}\sin\phi) + \Delta x_o' (R \sin\phi + L\cos\phi) \right] \tag{6}$$

where M_{13}=4.9 m is the momentum dispersion of this magnet, Δx_o is the beam position deviation at the entrance, $\Delta x_o'$ is the beam angular deviation. Comparing the two items in Eq.(6), the effect of angular deviation is more important. The effect of 1 mm of position deviation is: $\Delta p/p \sim 2.8 \times 10^{-5}$. The effect of 1×10^{-3} of angular deviation is: $\Delta p/p \sim 1.5 \times 10^{-3}$. Suppose the requisite energy calibration error $\Delta W/W \sim 1\%$, i.e., $\Delta p/p \sim 5.9 \times 10^{-3}$, the requisite

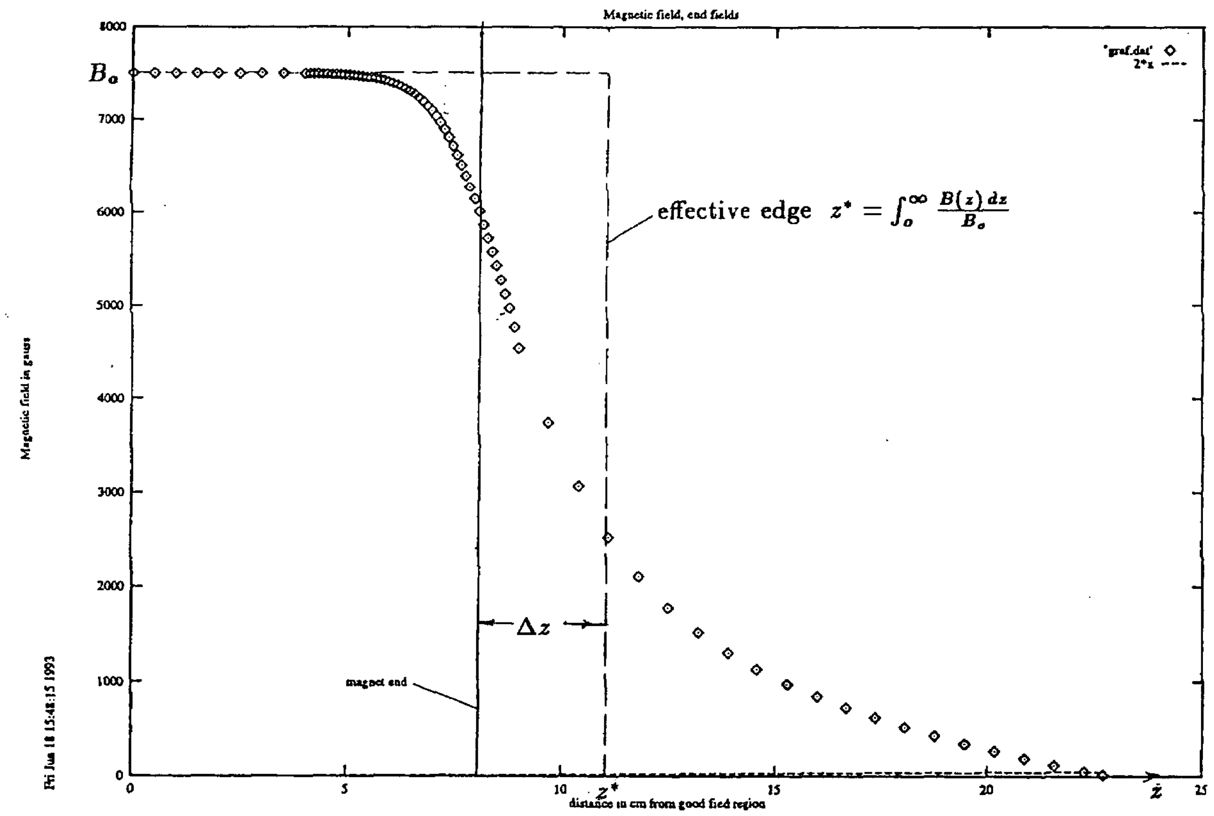

Figure 2: The magnetic field and effective edge

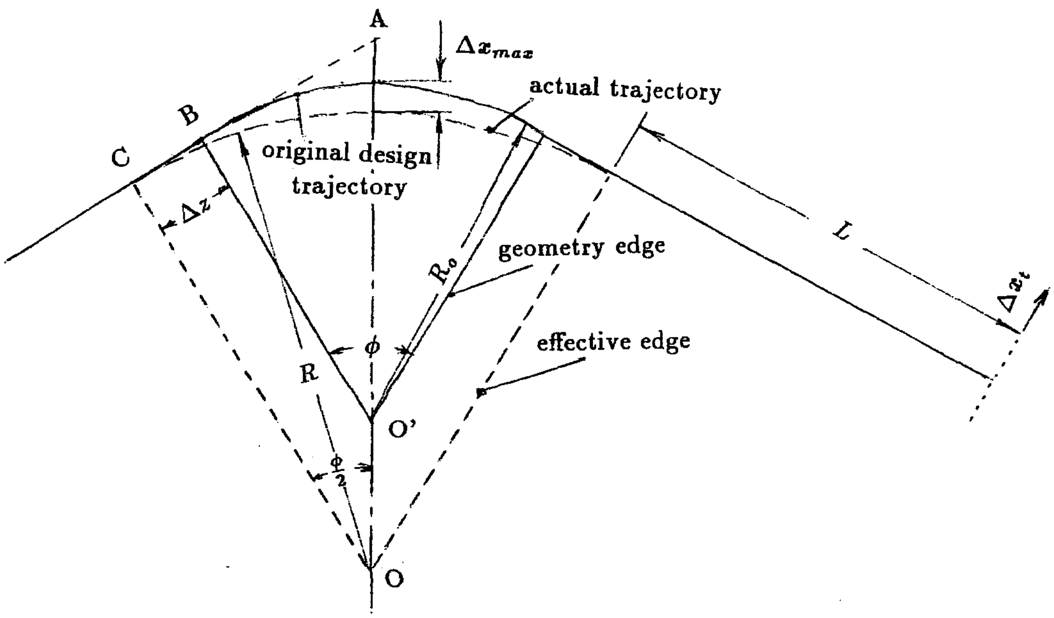

Figure 3: The effect of spraying field on trajectory.

$\Delta x'_o < 4 \times 10^{-3}$. It is not too difficult to get this accuracy by two beam position monitors. Anyway, in future beam centering must be ensured before an accurate measurement is done.

By the way the effective curved radius and effective edge should also be used in the calculation of beam transport.

2 The Resolution of the Energy Spectrometer

As known, for a normal design when disregarding the beam utilization efficiency, we only need to consider the X-direction motion. We have

$$x_2 = M_{11}x_1 + M_{12}x'_1 + M_{13}\Delta p_b/p_b \tag{7}$$

where x_2 is the measured half beam width at the measurement point W. Usually $M_{12} = 0$ is designed, i.e., the beam is focused at the measuring wire. In this case M_{11} is called the magnification. The M_{13} is called the momentum dispersion :

$$M_{13} = D = \frac{\partial x}{\partial \Delta} \tag{8}$$

where $\Delta = \Delta p_b/p_b$. $M_{13}\Delta p_b/p_b$ represents the horizontal distance of two particles of different momenta ($\Delta p_b = \Delta \times p_b$) with same x_1 and x'_1. At image point we have:

$$M_{13} = R\{1 - \cos\phi + \frac{l}{R}\sin\phi\} \tag{9}$$

The momentum resolution of the spectrometer is defined by:

$$R_p = \frac{\Delta p^*}{p^*} = \frac{M_{11}x_1}{M_{13}} \tag{10}$$

The condition of accurate measurement of the momentum divergence is:

$$R_p \ll \Delta p_b/p_b \tag{11}$$

This is the reason that an object slit is commonly used. According to the design result which is based on the theoretical emittance parameters, the half width, $M_{11}x_1$, is $\sim$4 mm. In our case, $R = 4.33\,\mathrm{m}$, $\phi = 40°$, l=6.1 m, i.e.,M_{13}=4.9 m. That is said, $R_p \approx 0.081\%$. Generally, the momentum divergence of linac of energy of 400 MeV is : $\Delta p_b/p_b \sim 0.15\%$.

From the above discussion, it can be concluded that under the focusing condition, what we measure is:

$$x_2 = M_{11}x_1 + M_{13}\Delta p_b/p_b \tag{12}$$

The beam image width ($M_{11}x_1$) may be of the same order as the energy divergence distance ($M_{13}\Delta p_b/p_b$), i.e. in order to get accurate result of $\Delta p_b/p_b$, a limitation object slit is needed, or the beam image width must be more accurately calculated as follows: (1) measure the emittance parameters at the position A; (2) calculate the requisite quadrupole current I_1 and I_2 for the focusing condition; (3) tune the quadrupole current to get a minimum beam width at the measuring wire W ; (4) measure the emittance at the position C at this condition; (5) calculate the beam width, $M_{11}x$, under the above parameters.

REFERENCE

[1] M.Popovic, LU-178.